A model of harp plucking

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In this paper, a model of the harp plucking is developed. It is split into two successive time phases, the sticking and the slipping phases, and uses a mechanical description of the human finger's behavior. The parameters of the model are identified through measurements of the finger/string displacements during the interaction. The validity of the model is verified using a configurable and repeatable robotic finger, enhanced with a silicone layer. A parametric study is performed to investigate the influence of the model's parameters on the free oscillations of the string. As a result, a direct implementation of the model produces an accurate simulation of a string response to a given finger motion, as compared to experimental data. The set of parameters that govern the plucking action is divided into two groups: Parameters controlled by the harpist and parameters intrinsic to the plucking. The former group and to a lesser extent the latter highly influence the initial conditions of the string vibrations. The simulations of the string's free oscillations highlight the large impact the model parameters have on the sound produced and therefore allows the understanding of how different players on the same instrument can produce a specific/personal sound quality.
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I. INTRODUCTION

The question of the different qualities of the sound played by different harpists is a subject of discussion among players and acousticians. While everybody agrees that a player can easily be recognized by his/her style and technique, skilled players insist on the possibility to identify each other from the sound only at the individual note level. Obviously, playing at the same plucking position on the string and producing the same global sound power, players can control some other aspects of the sound quality. Earlier results[1] obtained on 10 skilled harpists indicate that each of them provide to the string a highly repeatable plucking path depending on the playing technique and the finger studied. Besides, it has been shown[1] that the plucking position is almost the same for each harpist. It can therefore not explain the plucking specificity of the harpists. They finely control the initial shape, velocity, angle of polarization, and rotation they provide to the string before releasing it, resulting in an accurate control of its free oscillations and of the sound produced. However, the way the player controls the harp plucking is not yet understood on a physical basis. A better understanding of the mechanical parameters that govern the plucking would allow us to control sound synthesis of plucked string instruments in a realistic way. Indeed, although the numerous investigations of the physics of musical instruments allow the production of satisfying sound synthesis,[2,3] there is a lack of realism in the control of their initial conditions, i.e., the state in which the musician sets the instrument to produce a sound. This is mostly achieved by tuning parameters until a satisfactory sound is reached.

Most of the studies about the plucking action and its synthesis deal with the classical guitar.[4–9] The plucking action is described as "ideal" (Refs. 4–6), i.e., the string vibrations are initialized only through a displacement with no velocity.[10] Furthermore, the musician's touch is reduced to that of a plectrum, corresponding to a triangular initial shape of the string. However, the presence of the musician and his control on the note produced has been investigated for the classical guitar.[7–9] In these studies, physical modeling of the finger/string interaction has been proposed with parameters adjusted to produce the desired sound rather than physically relevant considerations. Experimentally based investigations of the concert harp plucking[11,12] has provided finger-string motion to estimate the mechanical parameters of the finger.[11,12] However, the experimental constraints do not allow to point them out in a robust manner. Therefore the estimation of relevant mechanical parameters to describe the plucking action remains a tricky issue. Besides, a study of the piano action mechanism[13] indicates that the viscoelastic behavior of the finger should be taken into account. Looking at the literature, a cautious investigation of the human finger behavior in plucking musical instruments has not yet been undertaken.

The present paper aims at modeling the classical concert harp plucking action. The proposed model is based on parameters estimated using measured displacements of the finger and of the string. The latter are expected to describe both the mechanical parameters specific to the harpist's finger morphology and the one she/he has the possibility to control during plucking. Their impact on the sound produced is also investigated to point out the set of parameters revealing the specific sound of a musician. A modeling of the finger/string interaction is provided in Sec. II. Then an experimental procedure is described in Sec. III to capture the finger's and the string's motion during the plucking action. On one hand, these measurements help to highlight the mechanical parameters of the musician's finger and on the other hand, they validate their relevance to model the string's response under a given finger's action in Sec. IV. Section V investigates the impact of these control parameters on the

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initial conditions of the string's free oscillations through a parametric study. Eventually, the parameters' estimation is applied in Sec. VI to plucking actions in real musical context derived from previous measurements.[1]

II. HARP PLUCKING ACTION MODELING

The harp plucking can be split into two successive time phases:[1] The sticking and the slipping phases. The harp plucking modeling is structured accordingly.

A. Sticking phase

1. Description

During the sticking phase, the finger pulls a segment of the string from its initial position up to the point where the tangential shear force exerted by the string on the skin reaches a threshold force F_{max}, controlled by the harpist. Assuming that the displacement of the finger's distal phalanx and the string displacement only take place in the plane, fixed to the harp, perpendicular to the strings,[1] we only investigate their trajectories in this plane referred to as $(x0z)$ in this paper. Their components are referred to as (x_s, z_s) and (x_f, z_f) in Fig. 1, respectively. As we only deal with isolated plucking actions, the string is considered to start from its rest position at t_c, i.e., $x_s(t=t_c) = x_0 = 0$, $z_s(t=t_c) = z_0 = 0$, at the beginning of the sticking phase. The mechanical behavior of the finger has to be taken into account to describe the sticking phase because the finger is squeezed while pulling the string. This deformation depends on both the string's and the finger's mechanical properties.

2. Skin's mechanical properties

Many studies have investigated the mechanical properties of human finger.[14–17] Whereas it is structured in three layers (the epidermis, the dermis, and the hypodermis) with various mechanical properties, it has been shown that the finger's response to an external load can be seen like a monolayer material with viscoelastic properties. Considering human tissue, Zener and Kelvin–Voigt viscoelastic models[18] are commonly used.[19,20] Furthermore, the finger's response to any load depends on the dynamic properties of the *stimuli*, as its force, velocity, magnitude, frequency, as well as the angle between the finger and the contact surface.[21] Eventually, viscoelastic models with non-linear components depending on the finger indentation are often used.[15,16,22–24]

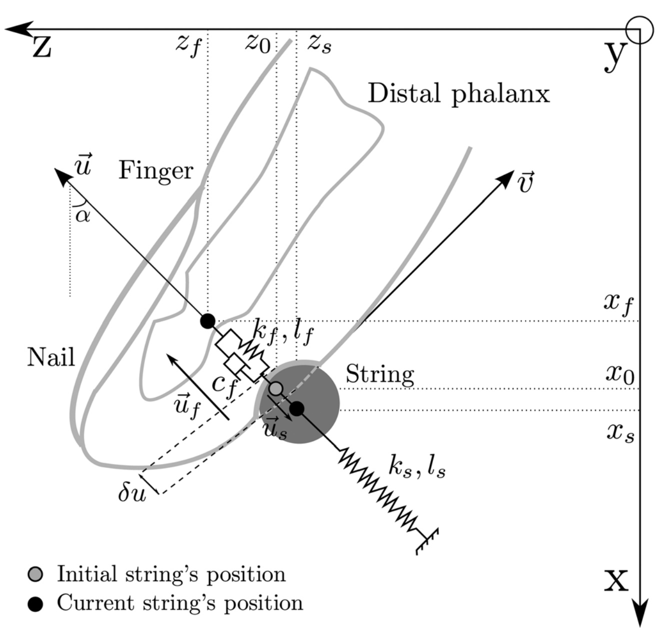

FIG. 1. Finger/string interaction during the sticking phase and its equivalent modeling.

3. Modeling

Figure 1 illustrates the interaction between the finger and the string during the sticking phase with its equivalent model. We model the sticking phase of the plucking action using a Kelvin–Voigt model. It consists in a spring and a dash-pot connected in parallel, reflecting the elastic and the viscous properties of the material. The spring's stiffness and equilibrium length are denoted k_f and l_f, respectively, while the damping of the dash-pot is referred to as c_f. The equilibrium length l_f represents the thickness of the finger at rest. On average, it is estimated at 1 cm for the forefinger. Besides, as this phase is quasi-static, we model the string as a single spring of stiffness k_s. Indeed, assuming that we have a flexible string of uniform linear density ρ_l, stretched to a tension T and fixed at its ends, its free oscillations velocity can be easily computed.[25] For instance, regarding the 30th harp string plucked at the third of its length and released with an initial displacement of $D_{t_r} = 5$ mm from its rest position and an initial velocity of $V_{t_r} = 2$ m/s, the maximal string velocity during the following oscillations is estimated at about 3 m/s. Because the string velocity during the sticking phase of typical duration 300 ms does not exceed 0.5 m/s, the latter is then assumed to be quasi-static. k_s is estimated based on the string's tension T, its length L, the plucking position y_0, and the width of the excitation Δl as

$$k_s = T\left(\left(y_0 - \frac{\Delta l}{2}\right)^{-1} + \left(L - y_0 - \frac{\Delta l}{2}\right)^{-1}\right). \tag{1}$$

Furthermore, the string equilibrium length l_s is chosen to be zero because the origin of the x and z axis is taken at the string's rest position.

The harpist's finger displacement in the $(x0z)$ plane is the input of the plucking action modeling. To model the response of the string (x_s, z_s) to this excitation, we define the frame of reference $(\vec{u}, \vec{v})$ related to the plucking action, where $\vec{u}$ and $\vec{v}$ are normal and tangential to the skin in the contact area, respectively. Figure 1 illustrates this frame of reference at a given instant of the sticking phase. The string's and the distal phalanx's displacements are referred to as u_s and u_f along the u axis, respectively. The latter is defined as

$$u_f = l_f - \sqrt{(z_f - z_0)^2 + (x_f - x_0)^2}, \tag{2}$$

and the plucking orientation α is estimated throughout the sticking phase as

$$\alpha = -\arctan\left(\frac{z_f - z_0}{x_f - x_0}\right). \tag{3}$$

These parameters will convey the string's motion in the ($x0z$) plane:

$$x_s = x_0 + u_s \cos\alpha, \tag{4}$$

$$z_s = z_0 - u_s \sin\alpha. \tag{5}$$

Under quasi-static hypothesis, the u-axis component of the force balance between the finger and the spring that models the string is written as

$$k_f(u_f - u_s - l_f) + c_f\left(\frac{\partial u_f}{\partial t} - \frac{\partial u_s}{\partial t}\right) - k_s u_s = 0. \tag{6}$$

We define the finger's indentation parameter δu (see Fig. 1), as

$$\delta u = l_f + u_s - u_f. \tag{7}$$

Equation (6) then writes

$$k_s u_s + k_f \delta u + c_f \frac{\partial \delta u}{\partial t} = 0. \tag{8}$$

The parameters k_f and c_f correspond to the elastic and the viscous characteristics of the finger. They need first to be estimated to determine the string motion u_s from Eq. (6). Although the relation between the load applied by a probe and the indentation has been shown to be exponential by some authors,[15,16] it appears that there is no clear agreement in the literature on the finger's stiffness and damping forms with respect to its indentation. Therefore parameters k_f and c_f are investigated under both the linear and exponential following forms:

$$k_f^{lin}(\delta u) = k_f^a \delta u, \quad c_f^{lin}(\delta u) = c_f^a \delta u,$$
$$k_f^{exp}(\delta u) = k_f^a e^{k_f^b \delta u}, \quad c_f^{exp}(\delta u) = c_f^a e^{c_f^b \delta u}. \tag{9}$$

The estimation of the parameters k_f^a, k_f^b, c_f^a, and c_f^b will be carried through measurements of the finger and string motion while plucking a string. This will be presented in Sec. IV.

B. Slipping phase

1. Description

In the final moments of the sticking phase, the harpist's finger turns around the string. Hence she/he defines the orientation γ of the slipping phase. At the beginning of the slipping phase ($t = t_s$), the string's position in the ($x0z$) plane is noted (x_{t_s}, z_{t_s}). From the beginning of the slipping phase until the release instant ($t = t_r$), the string slips on the finger's surface. The length δ_s of the slipping corresponds to the initial distance between the fingertip and the string, which is defined at $t = t_c$ by the harpist. During the slipping phase, forces occurring on the string's element are the restoring force $\vec{F}_{k_s}$, the friction force $\vec{F}_t$ and the normal force $\vec{F}_n$, see Fig. 2.

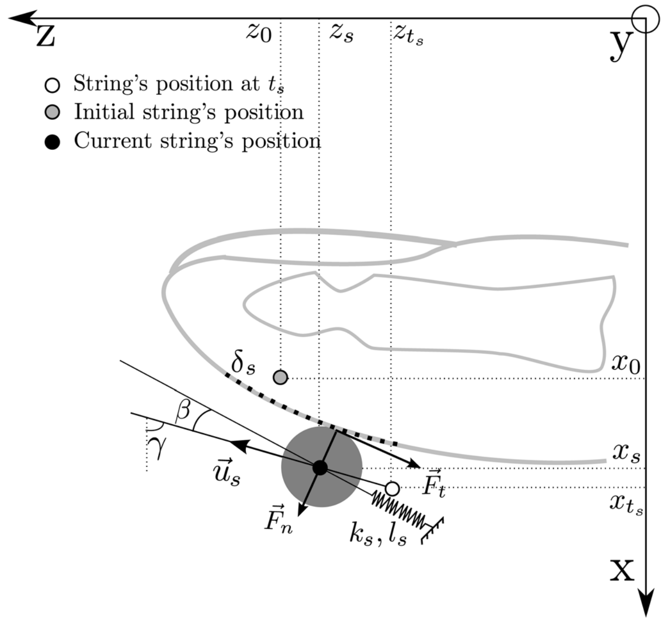

FIG. 2. Finger/string interaction during the slipping phase.

2. Friction properties of the human finger

The friction force governing this phase is investigated in the present paragraph. Note that the string's element contacting the finger is cylindrical and can be in gut, in nylon, or in steel as we focus on harp strings. In general terms, the friction of the human skin F_t is governed by[26]

$$F_t = F_a + F_d + F_v + F_r, \tag{10}$$

where

(1) F_a is the adhesive friction component related to the contacts of asperities between the finger and the contact surface,
(2) F_d is the friction related to the deformation of the finger,
(3) F_v is the friction due to capillary adhesion or viscous shearing, reflecting the self lubrication system of the finger,
(4) and F_r is the friction due to deformation of finger ridges.

Investigations of the forearm friction indicate that Eq. (10) can be reduced[27] to the terms F_a and F_d. However, it has been shown[26,28] that the friction related to the deformation of the finger F_d can be neglected relatively to the adhesive friction F_a. Therefore to model the string slipping over the finger surface during plucking, the friction force is assumed to be only described by the adhesive friction F_a. In the literature, it is written as a function the normal force F_n applied by the finger on the contact surface. Because of the viscoelastic properties of the human skin, the Coulomb model predicting a linear dependency of F_t in F_n through a friction coefficient μ has been questioned.[29,30] The non-linear model

$$F_a = \mu F_n^\lambda \tag{11}$$

has been proposed where λ is a coefficient lower than 1. However, a recent investigation[26] of the friction between human fingers and contacting surfaces indicates that a two-linear relationship exist between F_t and F_n, with the junction point at $F_n^{lim} = 1\,\mathrm{N}$:

$$\begin{cases} F_t = \mu_1 F_n, & \forall F_n \leq F_n^{lim}, \\ F_t = \mu_2 F_n, & \forall F_n > F_n^{lim}. \end{cases}$$

As the friction phenomenon in harp plucking occurs for normal forces always greater than 1 N,[1] we assume the relationship between F_t and F_n to be linear through a unique friction coefficient μ.

3. Modeling

Figure 2 illustrates the plucking action during the slipping phase. The direction of the slipping is given by the angle γ, which is controlled by the harpist. According to the Fig. 2, the string's motion in the $(x0z)$ plane writes as

$$x_s = x_{t_s} - u_s \cos\gamma, \tag{12}$$

$$z_s = z_{t_s} + u_s \sin\gamma. \tag{13}$$

Besides, as the string velocity can reach up to 2 m/s during the slipping phase,[1,31] the quasi-static hypothesis we used during the sticking phase can not apply during slipping. Hence the application of the Newton's second law to the string's element contacting the finger surface during the slipping phase writes as

$$\rho_l \Delta l \frac{\partial^2 \vec{u}_s}{\partial t^2} = \vec{F}_{k_s} + \vec{F}_t + \vec{F}_n, \tag{14}$$

where ρ_l is the string mass per unit length, or linear mass density. According to the Sec. II B 2, the previous the u-axis component of the Eq. (14) is

$$\rho_l \Delta l \frac{\partial^2 u_s}{\partial t^2} = \|\vec{F}_{k_s}\| \cos\beta - \mu\, \|\vec{F}_n\|, \tag{15}$$

where the amplitude of the normal force $\vec{F}_n$ is[32]

$$\|\vec{F}_n\| = \sqrt{(x_s - x_0)^2 + (z_s - z_0)^2}\, \frac{LT}{(y_0(L - y_0))}, \tag{16}$$

the amplitude of the restoring force $\vec{F}_{k_s}$ is

$$\|\vec{F}_{k_s}\| = \sqrt{(x_s - x_0)^2 + (z_s - z_0)^2}\, k_s, \tag{17}$$

and the angle β is written

$$\beta = \gamma - \arctan\frac{|z_s - z_0|}{|x_s - x_0|}. \tag{18}$$

Equation (15) then writes

$$\rho_l \Delta l \frac{\partial^2 u_s}{\partial t^2} = \left(k_s \cos\beta - \mu \frac{LT}{y_0(L - y_0)}\right) \times \sqrt{(x_s - x_0)^2 + (z_s - z_0)^2}. \tag{19}$$

Finally, the resolution of Eqs. (12), (13), (18), and (19) is performed through finite difference method, conveying the string's motion during slipping ($t_s < t < t_r$).

C. Implementation of the model

The sticking phase is mostly influenced by the finger's viscoelastic compression. Because the characteristic time of the sticking phase is long compared to the time period of the string oscillation, a quasi-static description is used. Therefore the contact force, *normal* to the skin in the contact area, can be deduced from the string displacement at the contact point. The slipping phase is triggered at the time when the *tangential* force exerted by the string on the finger reaches the maximum sticking force F_{max}, therefore when the string displacement in the tangential (skin surface) direction reaches the magnitude F_{max}/k_s. The force F_{max} depends on the normal contact force applied to the skin surface, which is related to the string displacement in the normal direction. Once the slipping phase has begun, the friction force reduces the natural string acceleration. As a consequence, by adjusting the initial contact position of the string on the finger and finger path during the sticking phase, the player can adjust the position where the slipping phase starts, the duration of the slipping phase, the position of the string release, the string velocity at release, as well as the initial polarization of the free string oscillation. Figure 3 proposes a block diagram of harp plucking modeling, including the mechanical parameters involved.

(1) The harpist's control parameters are the maximal force F_{max} applied by the finger on the string, as well as the length δ_s and the orientation γ of the slipping phase.
(2) The sticking and slipping parameters k_f^a, c_f^a, and c_f^b describe the contact between the finger and the string for a given plucking context (angle between the finger and the string, δ_s,...).

This model allows estimation of the finger mechanical properties from measurements of finger and string displacements during plucking action. In the following step, the model can predict the string's response to a given finger's distal phalanx motion (x_f, z_f). These two aspects will be discussed in Sec. V.

III. EXPERIMENTAL PROCEDURE

The finger/string interaction model proposed in this paper is compared to real finger and string motion during

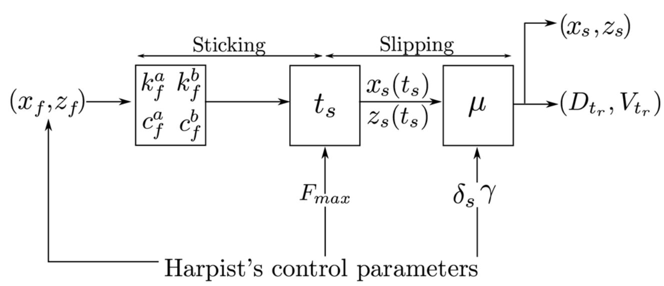

FIG. 3. Summary of the harp plucking modeling.

plucking actions. Three configurations are investigated. First, finger and string motion have been measured for plucking actions performed by an artificial finger shown in Fig. 4(a). It is a repeatable and configurable tool to pluck a string as desired. To model the human plucking, it is enhanced by silicone fingertips. We use cylindrical fingertips with a rounded ending and two different hardnesses. Fingertips are referred to as F1 or F2 in the following with F1 softer than F2. In addition, for variability issues, measurements have been performed three times with F2. The robot finger has been shown to reproduce accurately an input reference displacement and to produce a sound close to that of a real harpist's.[31,33] The use of this artificial finger is justified by its ability to provide a repeatable plucking with a planar motion, i.e., the closest to the model analysis. Then, to gradually investigate the robustness of the model, isolated plucking actions performed by a harpist are captured. She has been asked to pluck the 30th string eight times with the right forefinger as illustrated in Fig. 4(b). This second configuration represents an intermediate step between the robotic finger and harpist's in a real musical context because her plucking technique is more realistic than the former (for instance, with an additional rotation of the finger around the string) and does not contain the transitions' techniques between two succeeding notes. Eventually, finger and string motions have been measured for plucking actions performed by 10 harpists in various musical contexts as in arpeggio or chord sequences using the forefinger as well as the annular. The database used is the same as investigated in the previous description of the plucking action.[1] These measurements will help to point out the robustness of the model and to highlight tendencies in the whole set of mechanical parameters estimated according to the musical context.

The measurement protocol carried out is mostly based on capturing the motion of the finger and of the string with a high-speed camera set at 10 000 frames per second. As this experimental method has already been detailed in a previous paper,[1] we summarize here the main steps. The estimation of the finger and the string trajectories is performed by tracking markers, placed on finger and string at strategic places, through image processing.[1] More precisely, because we are interested in displacements referred to as x_s, x_f, z_s, and z_f in Fig. 1, markers are positioned as close as possible to the plucking position y_0 and to the nail, respectively. The latter is assumed to be rigid and to provide a good estimation of the distal phalanx displacement.

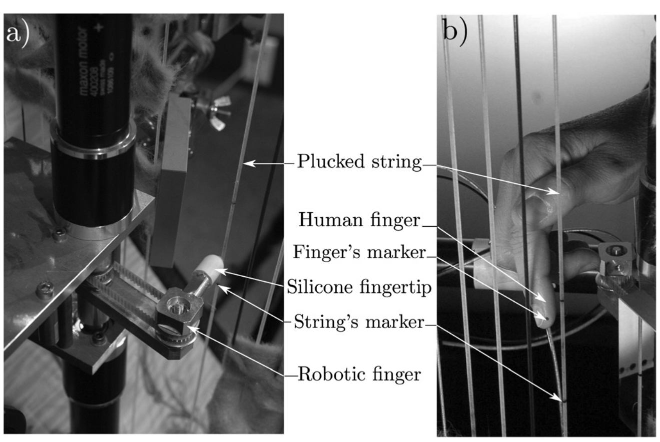

FIG. 4. Experimental setup using (a) robotized and (b) real plucking actions.

IV. PLUCKING PARAMETERS ESTIMATION

A. Sticking parameters

1. Method

The sticking parameters referred to as k_f^a, k_f^b, c_f^a and c_f^b are estimated using an experimental database of plucking actions (x_s, z_s) and (x_f, z_f). Using the latter combined with Eq. (2), Eq. (8) is solved with Runge–Kutta algorithm for a set of finger's stiffness and damping values. Then a wide range of values are tested through the Levenberg–Marquardt algorithm.[34,35] This allows determination by minimization of the best set of parameters to solve the equation. For this purpose, the experimental trajectories are previously approximated by a sixth order polynomial curve fitting.

The robustness of the method to input noise is investigated in Fig. 5 in the case of isolated notes played by the robotic finger. The reconstruction quadratic error is estimated as

$$\epsilon = \sqrt{\frac{1}{N}\left(\sum_{t=T_s}^{NT_s} |\tilde{u}_s(t) - u_s^{th}(t)|^2\right)}, \tag{20}$$

where T_s is the sampling period of the experimental data (10 μs) and $\tilde{u}_s$ the reconstructed string's displacement for a given finger and string (u_s^{th}) displacements over a wide range of artificial input noise added to the experimental data. The error is estimated for the four models investigated:

(1) Model EE: Exponential stiffness and damping (k_f^{exp}, c_f^{exp}),
(2) Model LE: Linear stiffness and exponential damping (k_f^{lin}, c_f^{exp}),
(3) Model EL: Exponential stiffness and linear damping (k_f^{exp}, c_f^{lin}),
(4) Model LL: Linear stiffness and damping (k_f^{lin}, c_f^{lin}).

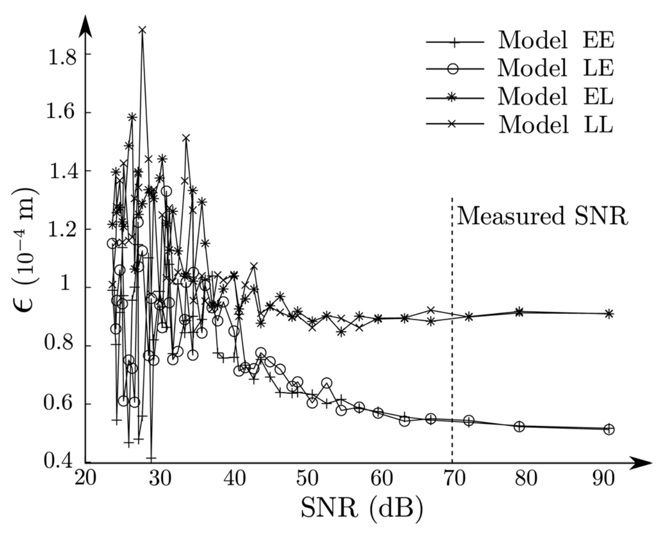

FIG. 5. Evolution of the reconstruction error of a reference curve versus its signal to noise ratio.

As expected, we observe that ϵ is higher for a low signal to noise ratio (SNR). Moreover, this error tends to become stable from a SNR above 60 dB. Because the typical SNR for our measurements is about 70 dB, we assume the method to be reliable. Eventually, the comparison of the four models indicate that a linear damping induces a higher reconstruction error ($\epsilon \simeq 0.9 \times 10^{-4}$ m) than an exponential one ($\epsilon \simeq 0.6 \times 10^{-4}$ m). Therefore models EE and LE appear to be the most relevant ones. Performances of EE and LE models for several different plucking situations show that the latter is more accurate and thus suitable than the former, suggesting that a linear stiffness and an exponential damping provide a better model.

2. Results

The database of the robotic plucking actions and the isolated notes performed by a harpist are used to determine the sticking parameters k_f^a, c_f^a and c_f^b for model LE. The reconstruction error estimated in percent for each plucking action is reported in Table I. It is computed with respect to the distance covered by the string during the plucking. First, we observe that the percentage error ϵ is always very small and that the reconstructions are better for the robotic plucking actions than for the human ones. This result is not surprising because the artificial finger performs a planar motion, whereas the harpist provides additional rotation to the string.

Then let us consider the robotic plucking actions in Table I. Three repetitions of the same motion have been performed with the fingertip referred to as F2. The estimated parameters corresponding are close; this tends to validate the parameter estimation process. Indeed, the variabilities around the mean values are, 3% and 13% for k_f^a, c_f^a and c_f^b, respectively. Eventually, the sticking parameters reflect the hardness character of the fingertip: The estimated values of k_f^a and c_f^a are greater for silicone fingertips with higher hardnesses. Thus for a given finger indentation, the finger/string interaction force has to be higher regarding F2 than F1. Besides, c_f^b reflects the maximal finger indentation reachable for a given fingertip. These estimations confirm that a softer material as F1 owns a larger range of possible indentations than F2.

Eventually, the sticking parameters obtained for the eight plucking actions performed by the harpist are investigated. Although the harpist used the same finger, we observe an important variability in the parameters estimations (about 40% around the mean values). It can be explained through variations in the contact surface. Indeed, as the harpist was asked to play isolated notes, she performed less repeatable plucking actions than in a realistic musical context. Then some plucking actions were for instance performed close to the fingertip, while others used the finger's pulp. Obviously, the mechanical properties of the finger vary along the distal phalanx, the fingertip being harder than the pulp. The estimation of δ_s partly supports this assumption since it is measured to be 0.2 mm for $P_{1,2,3,6,7}$, to be 0.04 mm for $P_{4,5}$, and 0.09 mm for P_8. Besides, the variability of the skin condition over the eight plucking actions and of the contact angle may explain these variations in the parameters' estimations.

TABLE I. Stiffness (k_f^a) and damping (c_f^a, c_f^b) coefficients estimated with the model LE and the percentage of reconstruction quadratic error ϵ according to the string's displacement for the whole set of robotic and human plucking actions.

	k_f^a (N m^{-2})	c_f^a (N s m^{-1})	c_f^b (m^{-1})	ϵ ($\times 10^{-6}$ %)
		Robotic finger		
F1	311	675	87	2.0
F2	339	1031	90	1.0
F2	326	1093	69	1.0
F2	350	1061	78	1.0
		Harpist		
P_1	1785	1480	84	20
P_2	1701	1754	72	22
P_3	1350	1867	89	8
P_4	294	659	6	6
P_5	1067	1134	78	3
P_6	714	475	139	5
P_7	788	516	51	12
P_8	1015	351	216	115

B. Slipping parameters

1. Method

The coefficient of friction μ is determined for the entire set of measured plucking action. For this purpose, the orientation of the slipping phase γ is first estimated through plucking action measurements[1] to about 45°. Then using measurements of the string displacements (x_s, z_s), Eq. (15) is solved using a Runge–Kutta algorithm for a given value of the friction coefficient. The Levenberg–Marquardt algorithm applied to this resolution with a wide range of friction coefficient values provide the more suitable one through minimization. As for the sticking phase, the experimental trajectories are approximated by a sixth order polynomial curve fitting before this estimation process.

2. Results

The human and the robotic plucking action databases are used to estimate their associated finger/string friction coefficients. The reconstruction error, computed following Eq. (20) for each estimation of μ, presented in Table II appears to be very small compared to the amplitude of

TABLE II. Friction coefficients (μ) and percentage of reconstruction quadratic error ϵ according to the string's displacement estimated for the whole set of robotic and human plucking actions.

	μ	ϵ ($\times 10^{-4}$ %)		μ	ϵ ($\times 10^{-4}$ %)
	Robotic finger			**Harpist**	
F1	1.00	1.8	P_1	0.99	0.4
F2	0.96	1.0	P_2	0.99	1.5
F2	1.00	0.2	P_3	0.97	5.0
F2	0.90	0.0	P_4	0.91	0.2
			P_5	0.87	0.1
			P_6	0.99	1.0
			P_7	0.97	4.3
			P_8	0.98	1.3

motion. The slipping phase modeling presented in Sec. II B 3 is therefore reliable. Furthermore, as expected, the variability observed within the eight plucking actions performed by a harpist is higher than within different silicone fingertips. In both cases, μ is of the same order of magnitude for all the plucking actions. Hence this result indicates that the silicone fingertip of the robot finger shows slipping properties close to that of a human finger.

However, the variability observed between the three estimations of μ for the three plucking actions repeated by F2 is not negligible: It seems difficult to estimate accurately the friction coefficient. Indeed, it highly depends on the experimental context as the skin condition (dry, wet, clean,…)[36] and the angle between the finger and the contact surface as well as the shape and the material of the contacting object.[37] In addition, the range of friction coefficients measured for the finger in various experimental configurations is very large, and no result about the friction of the finger with a spherical probe (the closer experimental context to ours) has been pinpointed in the tribology literature. Hence based on a recent review of experimental results for the friction coefficient of human skin,[37] results focusing on fingers sliding on various material surfaces and on spherical probe sliding on forearms indicate that the normal force applied by the probe and its material as well as its geometry have a great influence on the friction coefficient.

C. Reconstruction of isolated plucking

The parameters of the sticking and of the slipping phases estimated either for the harpist, or for the artificial finger are used to simulate plucking actions. Figures 6(a) and 6(c) present two selected sets of finger and string displacements measured for the harpist and the artificial finger, respectively. Taking the finger's displacement as input reference of the simulation, the modeled string's response is also drawn in solid line. It is computed according to the process presented in Sec. II C. In both cases, the global shape of the string motion is consistent with the measurement. However, a deviation appears in the orientation of the modeled and measured string along the path for the harpist plucking action and, to a lesser extent, for the robotic plucking action in Figs. 6(a) and in 6(c). A close observation of the curves indicates that for the harpist plucking action, the measured string does not follow the same orientation α as the finger during the plucking. This is most probably due to the rotation the finger applies to the string around its axis, inducing an erroneous estimation of α. As the robotic finger performs a perfectly planar motion, the estimation of the latter variable and the reconstruction of the string displacement are better. Moreover the slight deviation occurring Fig. 6(c) is probably due to the finger indentation, affecting the estimation of the orientation α. Therefore the latter appears to be a key variable to deduce accurately the string displacement (x_s, z_s) based on Eqs. (4) and (5). Figures 6(b) and 6(d) present the same results as in Figs. 6(a) and 6(c) but with an additional adjustment of the string orientation α during the sticking phase. The string's displacement reconstruction is obviously more accurate than previously. The motion investigated for the robotic finger shows a more sinusoidal shape than that of the harpist; this helps in minimizing the error in the reconstruction at the end of the sticking phase.

Eventually, due to the strong stability of the friction coefficient, the reconstruction of the slipping phase is more straightforward. Thus the simulated displacement of the string matches the measured one for both harpist and robotic plucking actions.

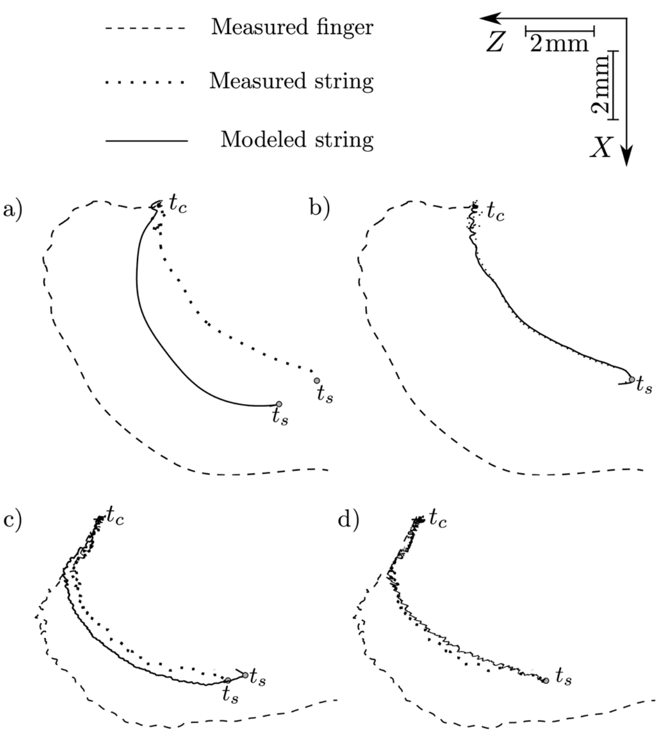

FIG. 6. Measured and simulated plucking action. (a) Plucking action performed by a real harpist finger, (b) with an additional adjustment of the string orientation, (c) plucking action performed by the artificial finger, and (d) with an additional adjustment of the string orientation.

V. INFLUENCE OF PLUCKING ON STRING OSCILLATIONS

A. Method

The influence of the plucking parameters on the string's oscillations are investigated in the present section. For this purpose, we input a finger's displacement (x_f, z_f) into the model and analyze the string's displacement (x_s, z_s) produced for a set of plucking characteristics (k_f^a, c_f^a, c_f^b, and μ), and control parameters (F_{max}, δ_s, and γ).

At the end of the sticking phase, the string's state can be described through its position $\vec{u}_{t_s}$ and its velocity $\vec{V}_{t_s}$. Because the string's trajectory will have a direction opposite to that of the finger during the slipping phase, its velocity is close to zero in every possible case. Hence the value of the string velocity at the end of the sticking phase is not expected to be a relevant parameter. However, as $\vec{u}_{t_s}$ is related to both the string's displacement relative to its rest position D_{t_s} and the slipping orientation, it is assumed to be of great importance relatively to the initial displacement D_{t_r}, velocity V_{t_r}, and angle of polarization γ at the beginning of the string's free oscillations. During the sticking phase, $\vec{u}_s$ is governed by the finger's mechanical parameters and by the threshold force for sticking the harpist applies to the string. Based on classical string's vibration theory, Eq. (16) provides the linear dependency between F_{max} and D_{t_s}. However, the relationship between the latter and k_f^a, c_f^a, c_f^b is not straightforward. It is investigated through the path followed by the string for a given finger's motion. In addition, according to the previous results, F_{max}, μ, γ, and δ_s may directly impact D_{t_r} and V_{t_r}.

A parametric study is carried out to point out the influence of this set of parameters on the initial conditions for oscillation D_{t_r} and V_{t_r}. Fixing the whole set of parameters but one to a reference value allows to investigate variations of D_{t_r} and V_{t_r} according to the reachable range of values of the unfixed parameters. Based on previous numerical or experimental estimations of the plucking parameters, the following ranges of the parameters are defined as:

(1) $k_f^a \in [10; 2000]\,\mathrm{N\,m^{-2}}$; $Ref = 326\,\mathrm{N\,m^{-2}}$,
(2) $c_f^a \in [500; 2000]\,\mathrm{N\,s\,m^{-1}}$; $Ref = 1093\,\mathrm{N\,s\,m^{-1}}$,
(3) $c_f^b \in [1; 100]\,\mathrm{m^{-1}}$; $Ref = 69\,\mathrm{m^{-1}}$,
(4) $F_{max} \in [1; 10]\,\mathrm{N}$; $Ref = 5\,\mathrm{N}$,
(5) $\mu \in [0.87; 1.0]$; $Ref = 0.99$,
(6) and $\delta_s \in [0.1; 2]\,\mathrm{mm}$; $Ref = 1\,\mathrm{mm}$.

Note that the influence of the slipping orientation γ is not investigated here because it mostly influences the initial angle of polarization of the string's oscillations, whereas we only consider the string's oscillation in one dimension. Finally, we evaluate the influence of D_{t_r} and V_{t_r} on the string's vibrations through classical spectral descriptors. They are computed on the string free oscillations simulation. The descriptors we use are often calculated on the radiated sound rather than on the string vibration. Even if the relationship between the vibration of the string and the radiated sound is not straightforward (it actually takes into account the soundboard mobility and the radiating properties of the instrument), we expect relative values of the descriptors to give an insight on the influence of the plucking conditions. For this purpose, as in Sec. II, the string is assumed to be flexible, of uniform linear density ρ_l, stretched to a tension T, fixed at its ends, and plucked at one third of its length. Hence the modal amplitudes A_n and B_n of the transverse vibrations are[25,38,39]

$$A_n = \frac{2D_{t_r}\sin(k_n y_0)}{k_n^2 y_0(L - y_0)} \tag{21}$$

and

$$B_n = \frac{2V_{t_r}\sin(k_n y_0)}{k_n^3 y_0(L - y_0)c}. \tag{22}$$

Eventually, the following set of descriptors is calculated for the different initial conditions of string vibration. Denoting by $f_n = nf_0$ the eigenfrequencies and f_0 the fundamental frequency,

$$CGS = \frac{\sum_n nf_0(A_n^2 + B_n^2)}{\sum_n (A_n^2 + B_n^2)}, \tag{23}$$

$$\sigma^2 = \frac{\sum_n (nf_0 - CGS)^2 (A_n^2 + B_n^2)}{\sum_n (A_n^2 + B_n^2)}, \tag{24}$$

where CGS is the spectral centroid, and σ^2 is the spread of the spectrum around CGS. The former, CGS is expected to show a good correlation with the sensation of brightness of the sound produced,[40] while σ^2 describes the spectrum's shape.

B. Results

The path followed by the string during the sticking phase is first investigated. Figure 7 presents seven graphs. Each of them presents the finger and string motion in dashed and dotted lines, respectively. They correspond to the second

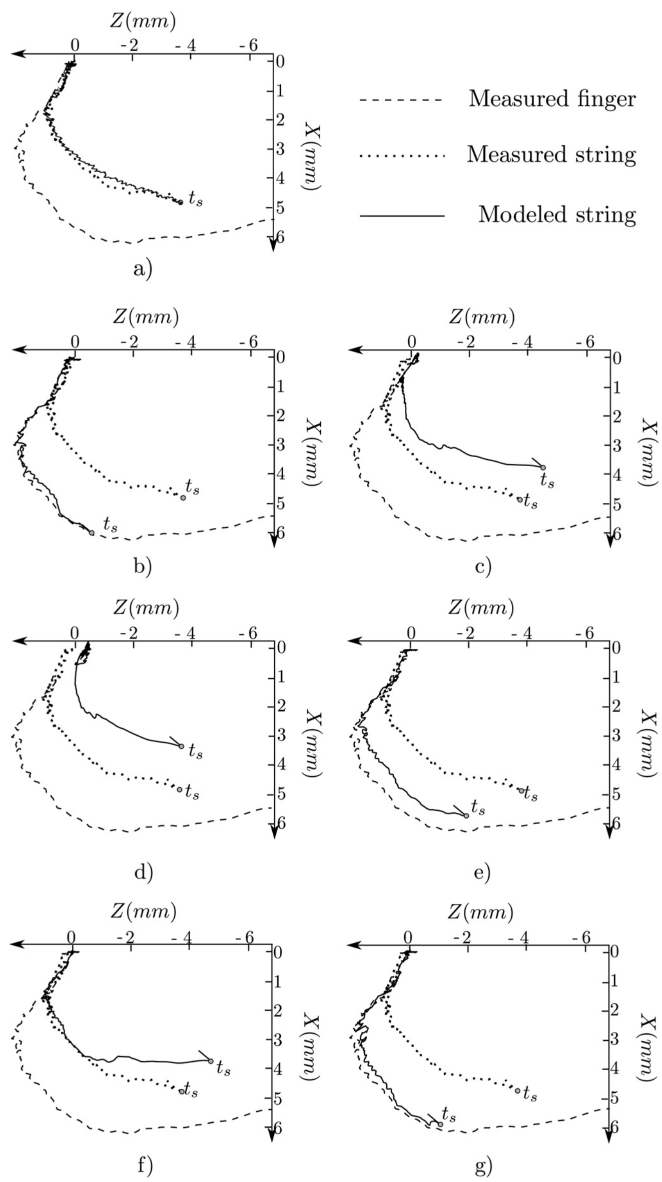

FIG. 7. Trajectories of the finger and the string estimated through measurements of a robotic plucking action with the A5-fingertip. They are associated to the modeled string trajectory in the $(x0z)$-plane for a large range of finger's parameters values. (a) Reference, (b) k_f^a: Minimum value, (c) k_f^a: Maximum value, (d) c_f^a: Minimum value, (e) c_f^a: Maximum value, (f) c_f^b: Minimum value, and (g) c_f^b: Maximum value.

measured plucking action performed by the artificial finger enhanced with F2. Figure 7(a) presents in solid line the simulated string's response to the finger motion with the mechanical parameters estimated previously in this paper. It is considered as a reference in the following paragraph. Figures 7(b) and 7(c) show the evolution of the simulated string's motion, while k_f^a takes its minimum and maximum value, respectively. Similarly, Figs. 7(c) and 7(d) on one side and Figs. 7(e) and 7(f) on the other side report the impact of c_f^a and c_f^b on the plucking modeling. The entire set of finger's characteristic during the sticking phase appears to have a non negligible influence on the string's motion. For instance in the particular case of the k_f^a minimum value in Fig. 7(b), as the finger's stiffness is rapidly compressed, the finger and the string follow the same trajectories. Besides of the plucking's shape, the position of the string at the beginning of the slipping phase is clearly related to the mechanical parameters k_f^a, c_f^a, and c_f^b, while the string's displacement D_{t_s} is mostly governed by F_{max}.

Table III reports the influence of F_{max}, δ_s, and μ on the initial conditions of the string vibrations D_{t_r} and V_{t_r}. The former is prone to important variations (from 0.03 to 7.4 mm) according to the maximal force applied by the finger on the string. It represents a variability of 211% relatively to its reference value (3.5 mm). To a lesser extent, initial conditions of the string oscillations are also impacted by the slipping distance δ_s with 43% of variation. As expected, the coefficient of friction does not influence the distance of the string relatively to its rest position at the release instant. As for V_{t_r}, the three slipping parameters have almost the same impact. They imply a variability of 93%, 133%, and 80% around its reference value (1.5 m s^{-1}). In addition, the behavior of V_{t_r} according to their variations is coherent. Indeed, both a higher slipping distance and a higher friction coefficient imply a longer slipping phase and a higher velocity at the release instant.

The influence of the initial conditions of the string's oscillations D_{t_r} and V_{t_r} on the spectral descriptors are presented in Table IV. The reference values used are 3.5 mm and 1.5 m/s for D_{t_r} and V_{t_r}, while the values investigated are in the ranges 0.03–7.4 mm and 0.1–3 m/s, respectively. The reported range of values reachable by D_{t_r} can imply a variation of 9 Hz in the spectral centroid, i.e., about 6% of the fundamental frequency of the studied string (D♭2 at about 140 Hz). It also impacts the spectrum's spread, which can reach up to nine times its smallest value. Although the impact of V_{t_r} is clearly less important than the one of D_{t_r} on the string's oscillations, it is not negligible. Indeed, it can induce a variation of 3% of the fundamental frequency in the spectral centroid, and the spread of the spectrum can reach up to 1.7 times its smallest value. Let us remark that these results, based on signal processing attributes, are clearly confirmed by informal listening to sound simulations of the string oscillation with the corresponding initial conditions.

TABLE III. Influence of the plucking parameters on the initial condition of the string vibrations. F_{max}, δ_s and μ are considered to vary from 1 to 10 N, from 0.1 to 2 mm, and from 0.87 to 1.0, respectively.

	F_{max} (N)	δ_s (mm)	μ
D_{t_r} (mm)	0.03–7.4	4–2.5	3.3–3.3
V_{t_r} (m s^{-1})	0.7–2.1	0.4–2.4	2.7–1.5

TABLE IV. Influence of the initial condition of the string vibrations D_{t_r} and V_{t_r} on the spectral descriptors CGS and σ^2. D_{t_r} and V_{t_r} are considered to vary from 0.03 to 7.4 mm and from 0.1 to 3 m/s.

	D_{t_r} (mm)	V_{t_r} (m/s)
CGS (Hz)	142–151	151–147
σ^2 (Hz)	374–3550	3703–2147

C. Discussion

This parametric study indicates first that the mechanical parameters governing the sticking phase in Fig. 2 have a great influence on the string's path during this phase. Hence, they impact the position of the string at the beginning of the slipping phase and consequently at the release time. This is of great importance relative to the initial angle of polarization of the string's oscillations and therefore to the sound produced. Then as for the slipping phase, the three parameters F_{max}, δ_s, and μ show a strong influence on the amplitude of the string vibration modes and the distribution of the energy on the string modes as function of the frequency. The values F_{max} and δ_s, which are directly controlled by the musician, appear to have the strongest influence.

VI. APPLICATION TO A MUSICAL CONTEXT

In the previous sections, we have restricted the analysis to isolated plucking actions performed by an artificial finger and by a harpist. The following section discusses the application of the model to actions performed in a real musical context. For this purpose, we use a finger/string motion database collected on 10 skilled harpists referred to as $H_{1\ldots10}$ performing either arpeggio or chord.[1] Only the plucking by the forefinger or the annular is analyzed. The parameters estimated for these plucking actions are reported Table V. There are no significant differences between the plucking positions of the different players because they all pluck the string at positions between about one-third and two-fifths of the distance from the soundboard to the neck. First, the variability estimated on the mechanical parameters describing the sticking phase is globally high: About 100% for c_f^b and and about 50% for k_f^a and c_f^a. Then regarding the parameters controlling the slipping phase, the variabilities are smaller but still non-negligible (about 15%). This indicates that these mechanical parameters are highly dependent on the plucking action, i.e., the harpist's control rather than on the harpist himself.

Because of the high variabilities, no clear result can be highlighted about the parameter c_f^a. However, k_f^a tends to be dependent on the playing technique. Indeed, higher values are computed while playing chord than arpeggio. This is illustrated for instance by harpists $H_{2,3,4}$ and to a lesser extent by harpists $H_{2,8,10}$. Concerning c_f^a, no rule can be extracted from Table V. Hence this would indicate that there

TABLE V. Influence of the plucking parameters on the initial conditions of the string vibrations. Note that some boxes are empty because some experimental data are missing or because of post-processing problems. Arp-Ann, Arp-For, Ch-Ann, and Ch-For referred to the four musical context investigated, i.e., arpeggio performed with the annular and the forefinger and chord performed with the annular and the forefinger.

Harpist	Parameter	Arp-Ann	Arp-For	Ch-Ann	Ch-For
	k_f^a (N m^{-2})	1504	1484	1611	1808
	c_f^a (N s m^{-1})	754	898	491	312
	c_f^b (m^{-1})	173	19	1	82
H_2	F_{max} (N)	8.2	6.3	7.5	5.7
	μ	1.2	1.3	1.0	1.3
	δ_s (mm)	0.7	0.5	0.7	0.5
	k_f^a (N m^{-2})	678	595	2635	1919
	c_f^a (N s m^{-1})	483	869	627	364
	c_f^b (m^{-1})	83	18	2	1
H_3	F_{max} (N)	4.4	3.9	5.1	4.5
	μ	1.2	1.2	1.1	1.0
	δ_s (mm)	0.6	0.8	1.5	1.5
	k_f^a (N m^{-2})	657	463	1408	826
	c_f^a (N s m^{-1})	647	640	471	644
	c_f^b (m^{-1})	88	38	30	
H_4	F_{max} (N)	7.3	3.7		6.7
	μ	1.3	1.0		1.5
	δ_s (mm)	1.2	0.8		
	k_f^a (N m^{-2})	716	1268	1939	
	c_f^a (N s m^{-1})	277	351	312	
	c_f^b (m^{-1})	10	14	73	
H_6	F_{max} (N)	7.0	4.5	6.8	
	μ	1.4	1.4	2.5	
	δ_s (mm)	1.2	1.0	1.3	
	k_f^a (N m^{-2})	605	1153	694	861
	c_f^a (N s m^{-1})	568	350	458	565
	c_f^b (m^{-1})		185	30	1
H_7	F_{max} (N)	3.7	3.9	2.5	2.7
	μ	1.2	0.8	1.2	0.9
	δ_s (mm)	1.1		0.2	0.4
	k_f^a (N m^{-2})	721	707	1281	758
	c_f^a (N s m^{-1})	908	546	618	694
	c_f^b (m^{-1})	165	1	120	9
H_8	F_{max} (N)	4.2	2.8	5.7	4.9
	μ	1.0	1.8	1.0	1.1
	δ_s (mm)	0.9	0.6	0.9	0.5
	k_f^a (N m^{-2})	317	613	1900	
	c_f^a (N s m^{-1})	702	664	525	
	c_f^b (m^{-1})	1	3	16	
H_9	F_{max} (N)	6.0		5.4	
	μ	1.1		1.2	
	δ_s (mm)	1.2		1.0	
	k_f^a (N m^{-2})	431	261	1814	887
	c_f^a (N s m^{-1})	346	942	369	493
	c_f^b (m^{-1})	1	1	38	22
H_{10}	F_{max} (N)	5.0	3.4	3.7	1.3
	μ	1.3	1.2	1.2	1.1
	δ_s (mm)	0.9	1.1	1.1	0.2

is no specific set of mechanical parameters relatively to a harpist but more probably to a plucking action.

Considering the slipping phase, the maximum force applied by the finger to the string appears to be higher while plucking with the annular than the forefinger. It is most likely explained by a compensation of the weaker control possible with the annular due to morphological reason. This result appears to be also related to the control parameter δ_s. For instance, considering harpists H_2 and H_4, the smaller δ_s, the smaller F_{max}. Furthermore, regarding the playing technique, trends seem to be specific to harpists. For example, harpist H_3 plays arpeggio with a smaller slipping distance than chord, independently of the playing finger, while δ_s is mostly specific to the finger for harpist H_8. These results are in agreement with previous ones highlighting that each harpist produces specific plucking actions relatively to the playing context. Furthermore, the playing context induces variations of the control that are bigger than variations amongst players, for one specific musical task. A global survey of the six parameters of the model that describe the plucking action from a mechanical point of view indicates that some of them are probably linked in the playing. For instance, when the player touches the string from a longer distance δ_s from the fingertip, it may be induced by the intention to play the note louder. The apparent correlation to a stronger sticking force F_{max} may come from the intention to play louder rather than on mechanical constraints. Therefore global playing indicators that lumping together several parameters of the model could be developed but ranges out of the scope of the present study.

VII. CONCLUSION

This paper has presented a model of the plucking action in the case of the concert harp. Measurements of the finger/string interaction have been carried out to determine the model parameters, and a parametric study provides the relevance of the model according to the string's free oscillations. The experimental setup was mostly based on the capture of the finger and the string motion in the plane perpendicular to the string's through a high-speed camera. The validity of the model is first discussed on ideal plucking actions performed by a configurable and repeatable robotic finger, enhanced with a silicone layer. Then the identification of plucking parameters, using the model, has been carried on isolated plucking actions performed by a real harpist and finally on plucking in real musical contexts.

The model for the finger/string interaction has been split into the two plucking action phases: The sticking and the slipping phases. During the sticking phase, the viscoelastic behavior of the finger is described using the classic Kelvin–Voigt model. The spring's stiffness and the damping of the dash-pot have been investigated as linear and exponential parameters depending on the finger indentation. The combination of a linear stiffness and an exponential damping has been shown to provide the most relevant modeling of the sticking phase. Subsequently, a consistent set of mechanical parameters can be extracted for the various silicone fingers as well as for the harpist fingers. Considering the slipping phase, the friction coefficient between the finger and the string is very tricky to determine accurately. This parameter depends on several variables that were not controllable in our measurements such as the contact angle between the finger and the string or the skin lubrication conditions. However, the values of the friction coefficients estimated for each experimental dataset are very similar, close to values found in the literature. Once the different plucking parameters of the model have been identified, the string and finger trajectories that are the output of the model lead to the initial conditions (displacement and velocity) of the free oscillation of the string. The influence of each plucking parameter has been investigated in relation to the initial conditions of the string vibrations. The mechanical parameters governing the sticking action have been shown to greatly influence the path followed by the string trajectory and position. The parameters the harpist controls directly while plucking are the force applied to the string, the slipping distance, and orientation. They highly influence the initial displacement and velocity of the string's free oscillations. Finally, the variations reported of the latter imply acoustically relevant differences in the string's oscillations. Therefore the mechanical parameters intrinsic to the harpist morphology and more specifically the control parameters strongly influence the sound produced. This justifies the musician's claim that they sound different at the individual note level depending on the way they put the instrument into vibrations. Differences in plucking are expected to induce changes in the spectral content of the sound, as well as on the sound level. Moreover, results indicate that the players produce quite different plucking actions according to the playing context. Those differences may be larger than the differences observed between players for a given musical task.

The rotation of the finger during the sticking phase requires more attention. Indeed, the motion of the harpist finger often shows a rotation of the phalanx before the beginning of the slipping phase that induces a change in the contact surface between the string and the finger as the finger gets more parallel to the string. It would then be valuable to develop a model in three phases, the sticking phase itself being analyzed in two successive steps: During the first one, the string being caught by the finger, controlling the string pulling, while during the second one, the string torsion and possible finger rotation control the triggering of the slipping phase. Furthermore, an experimental protocol dedicated to the contact between the finger and strings of several diameters and material, as found through the tessiture of the harp, would be of great interest. Finally, a perceptual test would be needed to confirm the influence of the mechanical parameters on the sound produced. Such a test, in relation to the player's technique and musical intention, may help to determine a combination of mechanical parameters that would give rise to global plucking parameters that are relevant from the point of view of the playing technique.

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