Experimental study of A0 and T1 modes of the concert harp

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String instruments are usually composed of a set of strings, a soundboard, and a soundbox with sound holes, which is generally designed to increase the sound level by using the acoustic resonances of the cavity. In the case of the harp, the soundbox and especially the sound holes are primarily designed to allow access to the strings for their mounting. An experimental modal analysis, associated to measurements of the acoustic velocity in the holes, shows the importance of two particular modes labeled A0 and T1 as it was done for the guitar and the violin. Their mode shapes involve coupled motions of the soundboard's bending and of the oscillations of the air pistons located in the sound holes. The A0 mode is found above the frequency of the lowest acoustically significant structural mode T1. Thus, the instrument does not really take advantage of the soundbox resonance to increase its radiated sound in low frequencies. However, contribution of mode A0 is clearly visible in the response of the instrument, confirming the importance of the coupling between the soundboard and the cavity. © *2007 Acoustical Society of America.*
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I. INTRODUCTION

The harp is one of the oldest string instruments. Its evolution from the prehistoric instrument to the modern concert harp led to the elaboration of constitutive elements, which are designed to efficiently radiate the sound. The modern concert harp is composed of a set of strings directly connected to a long thin flat soundboard attached to a fairly solid soundbox with several sound holes. These three elements are coupled in a complex manner and are attached to a base, a pillar, and an arm as shown in Fig. 1.

In a string instrument, the mechanical characteristics of the strings define the note to be played and the soundboard is designed as a sound radiator. Unfortunately, this sound radiator is not efficient in the low-frequency range when the acoustic wave length is greater than the size of the soundboard. An acoustical resonator, called the soundbox, is generally added in order to increase the sound level. The first acoustic resonance of the cavity can be used to reinforce the sound radiation of the instrument. This effect is used in the design of bass-reflex enclosures. The acoustical resonator has been the subject of many studies[1] on the guitar and on the violin: the acoustic motion inside the cavity interacts with the motion of the soundboard to produce two coupled modes. The first mode is called the plate mode and is associated with a strong bending motion of the soundboard. In the case of the guitar[2] and of the violin,[3] this mode is commonly labeled T1. The second mode is called the Helmholtz mode or A0 air mode and corresponds to a strong motion of an air piston located in the hole. The A0 air mode contributes to a significant increase of the sound radiation in the low frequency range. In order to well understand this low-frequency behavior of the guitar or of the violin, simple discrete models[4–6] have been carried out. The parameters of these models can be obtained from transfer functions measurements on these instruments.

In the case of the harp, the cavity and holes' sizes and shapes are not particularly designed to amplify the sound in the low-frequency range. One of the main reasons for the choice of sizes and locations of the holes is the facility for string mounting. The acoustic role of the holes is not well understood because the harp, and especially the soundbox, has not been the subject of many vibroacoustic studies.

The first study[7] was carried out on the small harp of Scotland. Modal analysis has been performed on the soundboard at different steps of its manufacturing. Air resonances were also investigated in the soundbox alone by burying it into sand in order to damp wall vibrations. No evidence of the presence of a Helmholtz resonance was found. Moreover, in playing configuration, the relationship between vibration modes of the instrument and radiated sound was not investigated. This study was later carried out on a Spanish harp of the baroque period,[8] close in size to the current concert harp. In this study, it was found that vibroacoustic interactions between soundboard vibrations and the acoustic motions of the air cavity lead to two coupled modes (112 and 146 Hz) having similar shapes and corresponding to A0 and T1 modes, respectively. This kind of result was also found on an unstrung Salvi Orchestra Concert Harp[9] by using holographic interferometric analysis of the soundbox. The author of this last study identifies A0 and T1 modes by measuring

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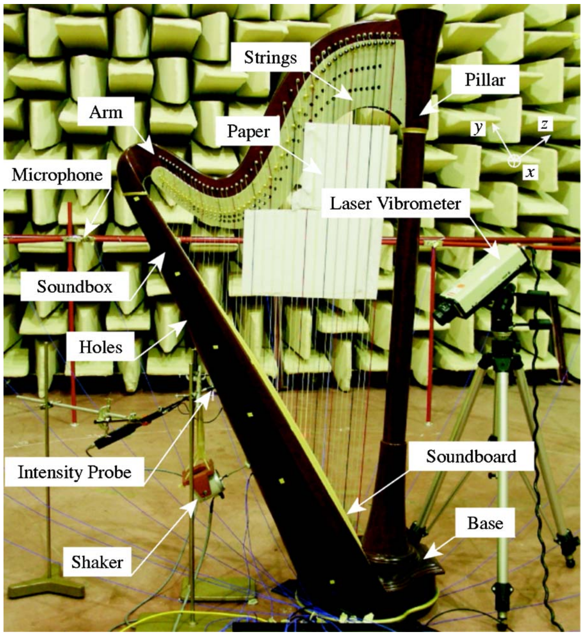

FIG. 1. (Color online) Experimental setup.

the changes of the structural response when sound holes are closed. For the Salvi Orchestra's soundbox, another study[10] has confirmed that if wall vibrations are damped by sand, the Helmholtz mode is clearly present in the acoustic response. A semi-empirical formula was proposed to predict its eigenfrequency. Another conclusion by Bell[9] is that one of the two coupled modes' (A0 and T1) presence in the response weakens when the soundboard is stressed by the strings. Thus, for a strung harp, the A0 mode is particularly difficult to identify. The reason for that is not clear. This difficulty was also pointed out on a Celtic harp.[11]

The aim of this paper is to identify the A0 air mode for the concert harp and to investigate the importance of its contribution to the instrument's response. For this purpose, the paper is divided into two parts. A study of the response functions of the instrument is first performed through the experimental modal analysis of the instrument's body and through an investigation of the acoustic field in the cavity. Then, the identification of the A0 and T1 modes is achieved by studying a modified instrument.

TABLE I. Dimensions of the five elliptical sound holes. The two dimensions correspond to the major axis and minor axis of each ellipse.

No.	Major axis (cm)	Minor axis (cm)
1	16.6	4.8
2	17.2	5.6
3	17.7	7
4	18.1	8
5	18.5	9.3

II. EXPERIMENTAL STUDY OF THE CONCERT HARP

A. Experimental procedure

The vibroacoustic behavior of a concert harp is experimentally investigated. All measurements are performed on an *Atlantide Prestige* concert harp lent by a French harp maker, Camac Harps. A schematic diagram is proposed in Fig. 2 with the principal dimensions of the instrument. The soundbox of the studied concert harp consists of a 6-mm-thick semi-conical shell with a total volume of the enclosed air of 0.029 m^3. On the back of the soundbox, there are five elliptical sound holes whose dimensions are shown in Table I. The concert harp is studied in playing configuration: all strings are mounted and tuned. For these measurements, strings are damped with paper to prevent their vibration while keeping the static deformation and load imposed by them on the soundboard. So, the string modes, including

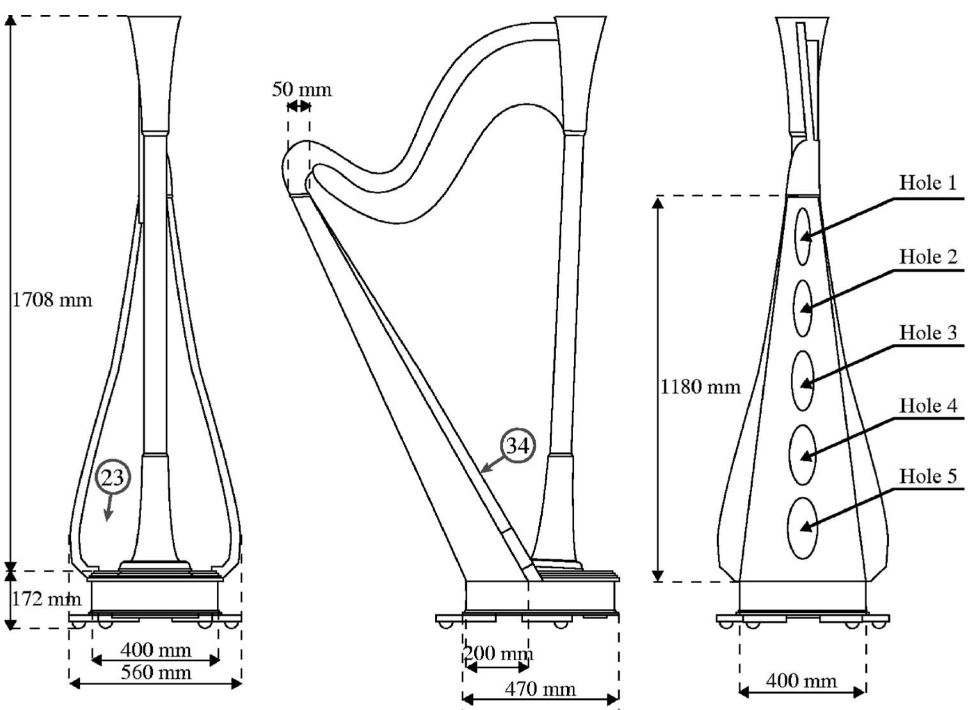

FIG. 2. Schematic diagram with dimensions of the *Atlantide Prestige* concert harp. The locations of two characteristic points 34 and 23 and hole number are also shown.

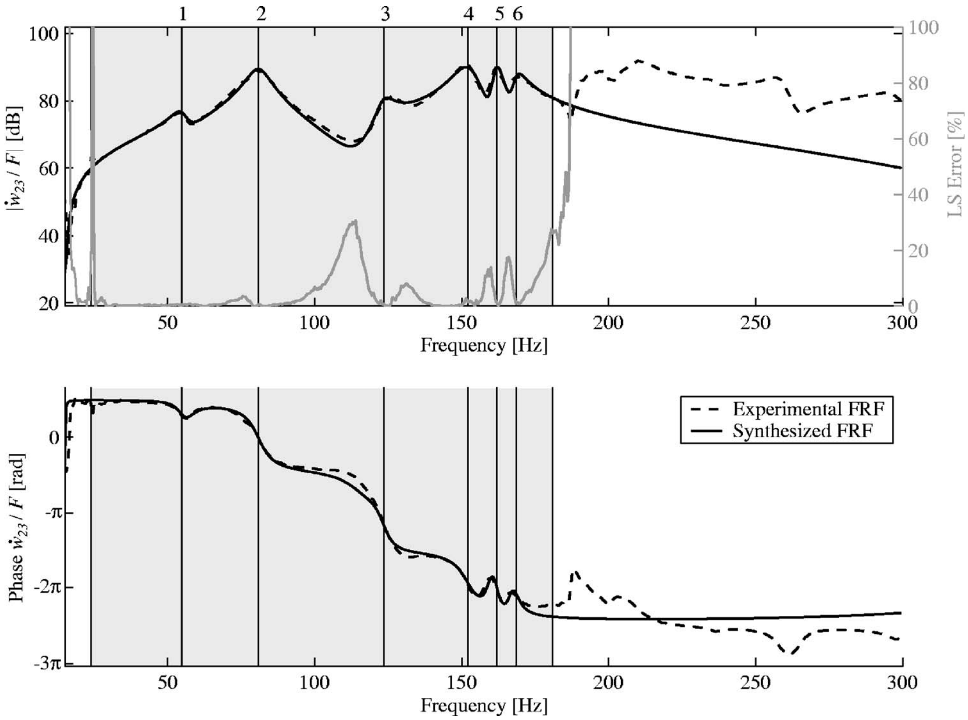

FIG. 3. Measured FRF $\dot{w}_{23}/F$, synthesized FRF $\dot{w}_{23}/F$, and least square error ϵ are shown versus frequency (Ref. 1 dB: 5×10^{-8} m s^{-1} N^{-1}). The grayed area corresponds to the frequency range in which modes have been identified. Numbers associated to vertical lines indicate the modal frequencies given in Fig. 4.

sympathetic modes,[12] are highly damped and are not evident in the instrument's response.

The experimental setup is shown in Fig. 1: the instrument is excited by a shaker driven by a white noise connected via a rod, through sound hole 4, to the back of the soundboard. The excitation force F is measured with an appropriate force sensor directly glued to the back of soundboard. The excitation point is labeled 34, as shown in Fig. 2, and is located between the Ab and the Bb string (respective fundamental frequencies at 103.8 and 116.5 Hz) attachment points. The vibratory velocity $\dot{w}$ is measured with a laser vibrometer. The acoustic velocity V in the middle of the sound holes is measured with an intensity probe. The far field acoustic pressure P is measured with microphones placed around the concert harp. Frequency response functions (FRFs) $H=\dot{w}/F$, $H_V=V/F$, and $H_P=P/F$ are then computed by a standard analyzer.

B. Experimental modal analysis of the instrument's body

The identification of structural modes of the soundbox in the low-frequency range is carried out by modal testing: eigenfrequencies, mode shapes, and damping parameters can be extracted from response functions measured at different points of the structure. The experimental mesh is composed of 60 points on the soundboard and of 18 points on the curved surface at the back of the harp, as shown on each modal shape in Fig. 4. The laser vibrometer is adjusted to measure the normal velocity on the soundboard. For each point on the curved surface, both the velocity along the z axis and along the x axis, defined in Fig. 1, are measured. Measurements are performed at each mesh point in the frequency range 0–300 Hz. A typical example of the measured frequency response functions is shown in Fig. 3.

The modal identification is carried out using the least square complex exponential method[13] implemented in the LMS software. Only six consecutive modes in the frequency range 24–181 Hz are identified because of the high modal density above 181 Hz as shown by the typical measurement at point 23 ($H_{23}=\dot{w}_{23}/F$) in Fig. 3. In this figure the synthesized response function ($\hat{H}_{23}$) and the least square error ϵ, defined by

$$\epsilon=\frac{|\hat{H}_{23}-H_{23}|^2}{|\hat{H}_{23}|^2}, \quad (1)$$

are plotted in order to validate the modal identification. According to this indicator ϵ, a good agreement between the measurement and the model can be found. Parameters obtained from this modal analysis are shown in Fig. 4. The following conclusions can be drawn for each identified mode.[14]

(i) Mode 1 has no nodes on its mode shape: the modal displacement is close to a global motion of the body depending on its connections to the arm and to the bottom of the pillar.

(ii) Modes 2 and 3 have common characteristics: The axial profiles of soundbox's displacements are similar to the first two mode shapes of a simply supported free beam. Note that as for mode 1, the shapes of modes 2 and 3 do not induce a change in the volume of the cavity: a weak coupling of these modes with the fluid inside the cavity can be expected.

(iii) Modes 4 and 6 have very similar mode shapes. The soundboard's displacement field corresponds to the first bending mode of a quasi-clamped plate. A slight

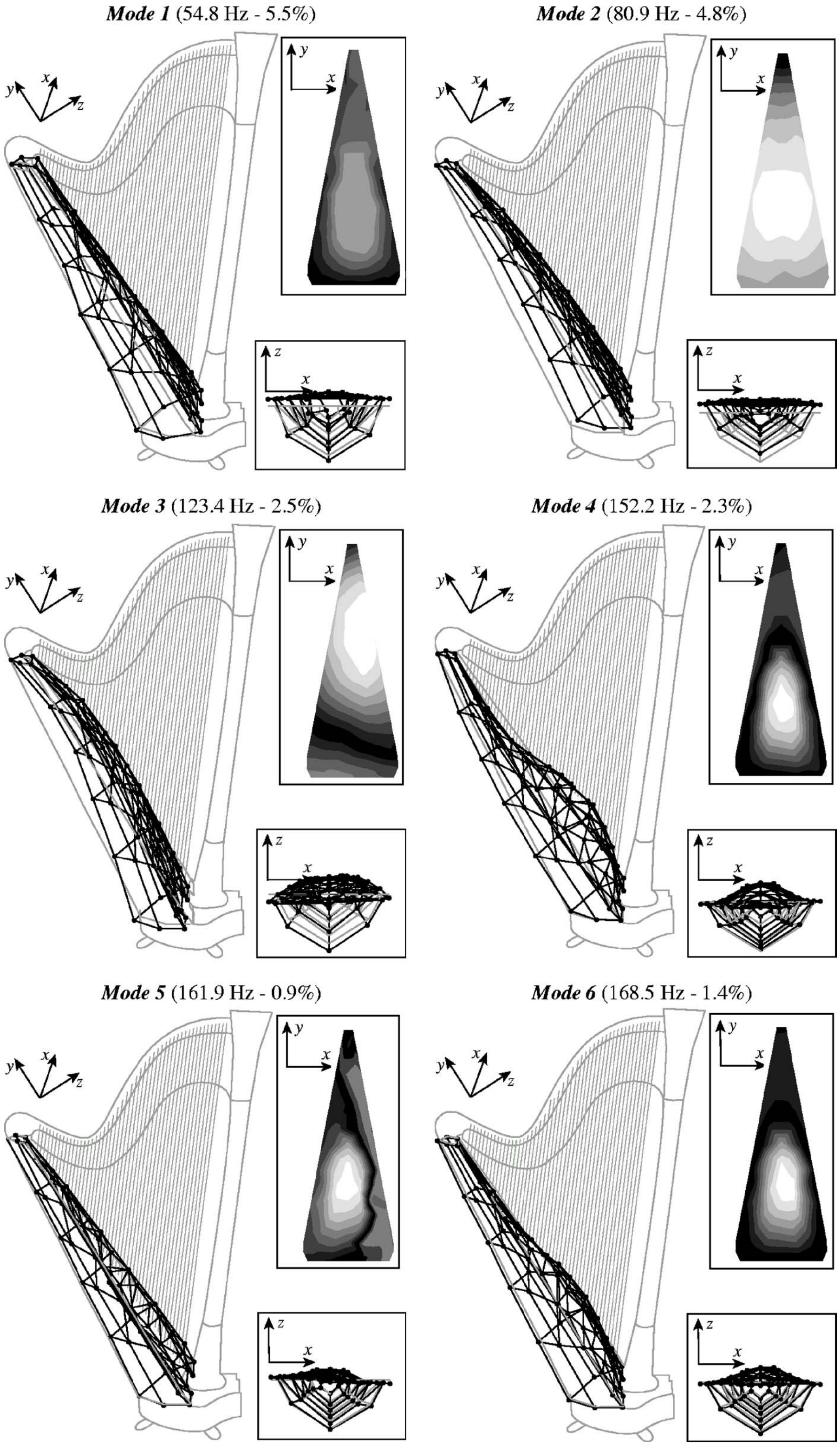

FIG. 4. Eigenfrequencies, damping coefficients and mode shapes of identified modes.

breathing motion of the soundbox is also observed. Shapes of modes 4 and 6 lead to an important change in the volume of the cavity.

(iv) Mode 5 is a pitch mode. In the measured response functions, this mode is not clearly present. It is actually not well excited since the shaker is connected close to the central line of the soundboard, which exactly corresponds to its nodal line. Since the strings are also attached on this nodal line, the role of this mode is not important when the instrument is played. For this reason, it will not be considered afterwards.

The two modes 4 and 6, which have similar shapes, have also been found on an unstrung concert harp[9] and on a strung Spanish harp.[8] However, when the harp is strung it seems difficult[9] to extract these two similar mode shapes. Nevertheless, in our study, these two modes were found in the playing configuration. Moreover, it should be noticed that the dis-

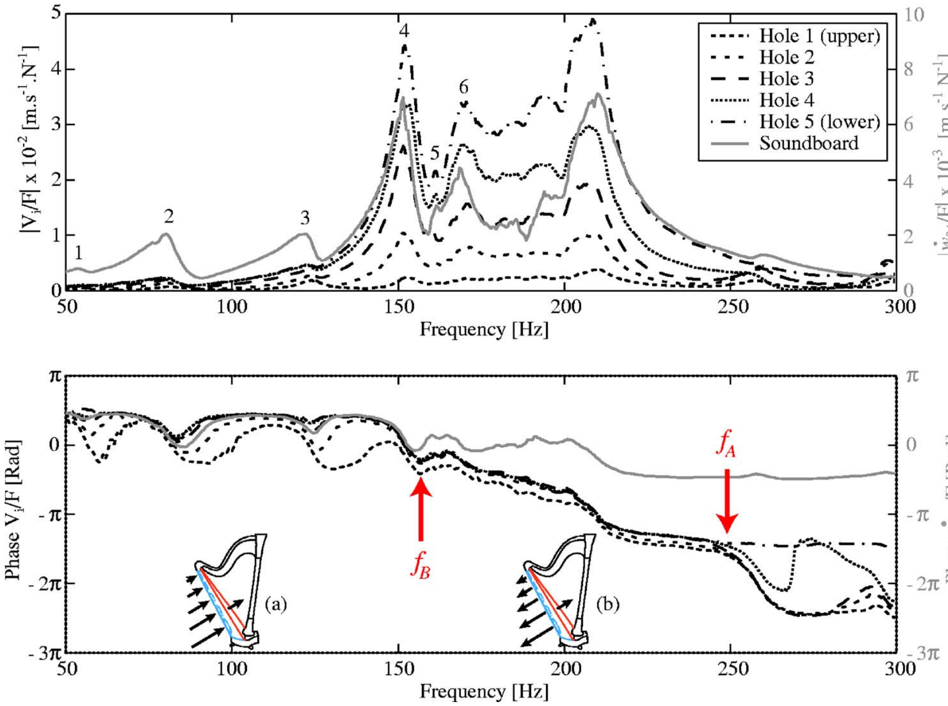

FIG. 5. (Color online) Magnitude and phase of FRF V_i/F (V_i: acoustic velocity in hole i) and of $\dot{w}_{34}/F$ ($\dot{w}_{34}$: velocity at point 34 on the soundboard). Note that magnitude scales for these two kinds of FRF are different. Numbers indicate the modal frequencies given in Fig. 4.

placements of the soundbox have the same order of magnitude as those of the soundboard; this is unexpected because the cavity seems to be much more rigid than the soundboard. Such a result was already mentioned for a Celtic harp.[15]

C. Analysis of the acoustic response functions of the instrument

In order to characterize the acoustic field inside the soundbox, the acoustic velocity in each hole has been measured in the low-frequency range 50–300 Hz, as shown in Fig. 5. Measurements are performed using the two microphones of an acoustic intensity probe. After an accurate calibration of the microphones, the acoustic velocity can be computed from the pressure measured at two points close to each other. Each sound hole can be described as an air piston of which the velocity is measured. It is found that the lower the hole is, the higher the magnitude of the velocity of the piston will be. Since the level of acoustic velocity for the upper hole 1 is far smaller than that of the four others, it cannot be considered as significant and will be ignored afterwards. So, in the studied frequency range the four other air pistons are found to be in phase below f_A (=250 Hz) and are no longer above. Thus, these measurements show that the acoustic field inside the cavity is mostly governed by the first acoustical mode below f_A=250 Hz. Above this particular frequency, other acoustical modes like longitudinal or pipe modes are present.

The mobility at excitation point 34 of the soundboard $\dot{w}_{34}/F$ is also plotted in Fig. 5. Its phase can be compared with the V_i/F phase, V_i being the acoustic velocity measured in hole i. It is found that below a second characteristic frequency, f_B (=160 Hz), the soundboard and all air pistons are in phase. Above this frequency f_B, but below f_A, the phase difference between FRF $\dot{w}_{34}/F$ and FRF V_i/F increases from 0° to 180°. This shows that in the frequency range f_A–f_B, the soundboard and the air pistons are out-of-phase. These particular phase relationships are schematically represented in Fig. 5 by arrows in harp drawings (a) and (b). The direction and length of the arrows that are plotted in these diagrams represent the phase and the magnitude of the velocity of the soundboard and of the air pistons below and above f_B.This result had already been found on another Camac concert harp in a previous paper[16] where the characteristic frequency f_B was found to equal 175 Hz.

In order to find out the implication of the acoustic field inside the soundbox on the acoustic far field of the instrument, we investigate the acoustic pressure around the concert harp. The pressure is measured in an anechoic room by 32 microphones regularly placed around the harp on a 2.35 m radius circle at 1.2 m in height. In the frequency range 50–220 Hz, the directivity patterns are found to be nondirectional, as shown in Fig. 6 for two selected frequencies corresponding to the eigenfrequencies of modes 4 and 6. The acoustic transfer function P_C/F measured in front of the harp (at the point labeled C defined in Fig. 6) is also shown in Fig. 7. As for afterwards measurements, the shaker used for the excitation is connected exactly on the central line of the soundboard and the acoustic effect of the pitch mode is then canceled out. In Fig. 7, we note that for a same force applied by the shaker, the acoustic pressure is much more important in the range 140–230 Hz than in the rest of the studied frequency range. Therefore, in a playing configuration, the harp seems to radiate the sound more efficiently in the range 140–230 Hz. The first two important peaks of acoustic pressure correspond to the eigenfrequencies of modes 4 and 6. Moreover, modes whose eigenfrequencies are above 200 Hz cannot be individually distinguished and their contribution to the response below 200 Hz is probably not negligible.

To conclude, six structural modes have been identified in the low-frequency range. Among these six modes, two play

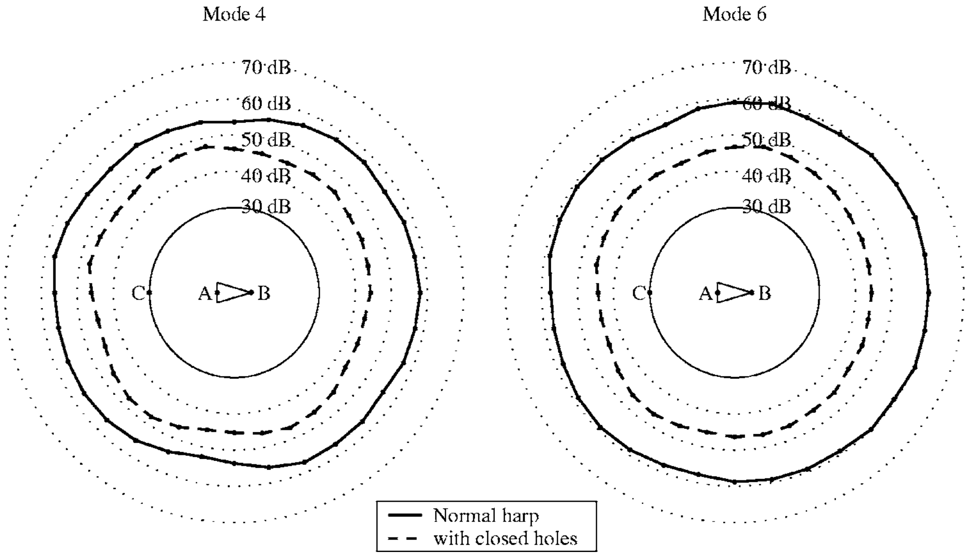

FIG. 6. Directivity patterns for modes 4 and 6 in two configurations: normal and with all holes closed by stoppers. Points A and B correspond to the bottom of the pillar and to the top of the soundboard respectively. Point C is the measurement point in front of the harp.

an important part in the sound radiated by the concert harp. They lead to a strong acoustic radiation, associated to a non-directional directivity pattern.

III. IDENTIFICATION OF A0 AND T1 MODES OF THE CONCERT HARP

A. Frequency response functions of a modified instrument

Considering only frequency response measurements on the instrument's body, modes 4 and 6 have similar mode shapes (see Fig. 4). However, the air piston motions are different for these two modes. To identify the nature (A0 or T1) of modes 4 and 6, the study of frequency response functions of a slightly modified instrument is performed. Mobilities measured at point 34 on the soundboard for three different configurations are compared with the normal configuration. The amplitude and frequency shifts of peaks are shown in Fig. 8 and in Table II.

The first modification consists of closing the sound holes of the concert harp as shown in Fig. 9 and labeled (1). Those are closed by using stoppers made with small tar plates. This configuration prevents all fluid motions inside the sound holes. This modification has heavily affected the instrument. Eigenfrequencies of modes 1, 2, and 3 undergo a shift of approximately −2 Hz due to the additional mass loading induced by the stoppers. Two additional peaks are seen below 200 Hz and in the rest of the frequency range the level is lower than in the normal configuration. The peak for mode 6

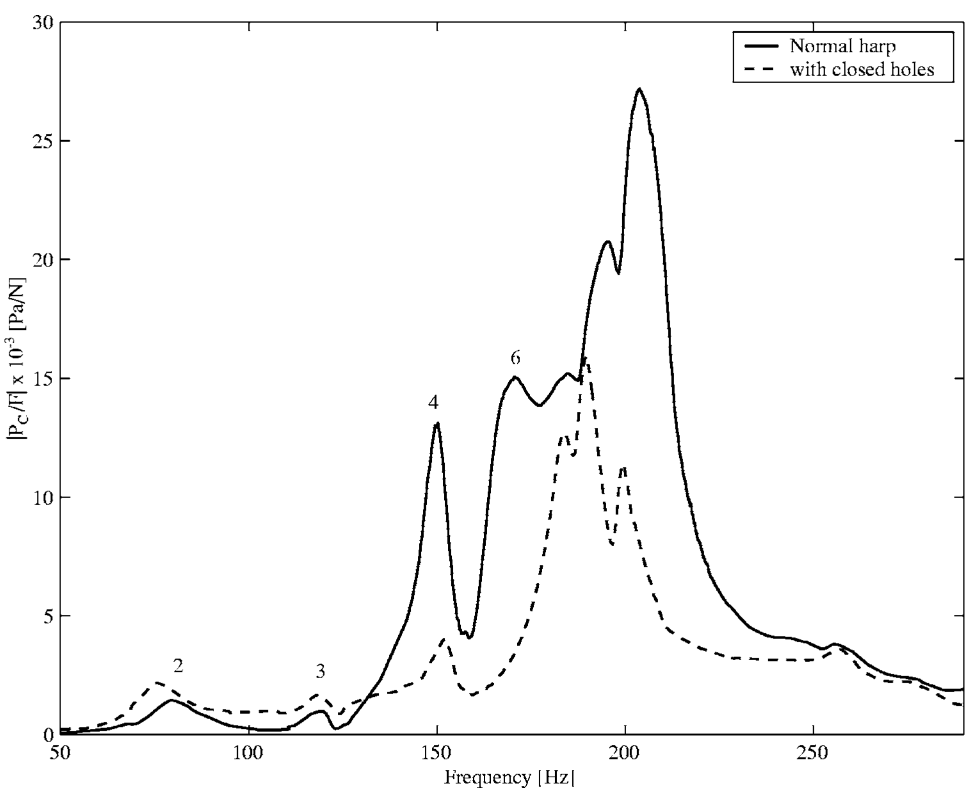

FIG. 7. Frequency response function at point C defined in Fig. 6. Numbers indicate the modal frequencies given in Fig. 4.

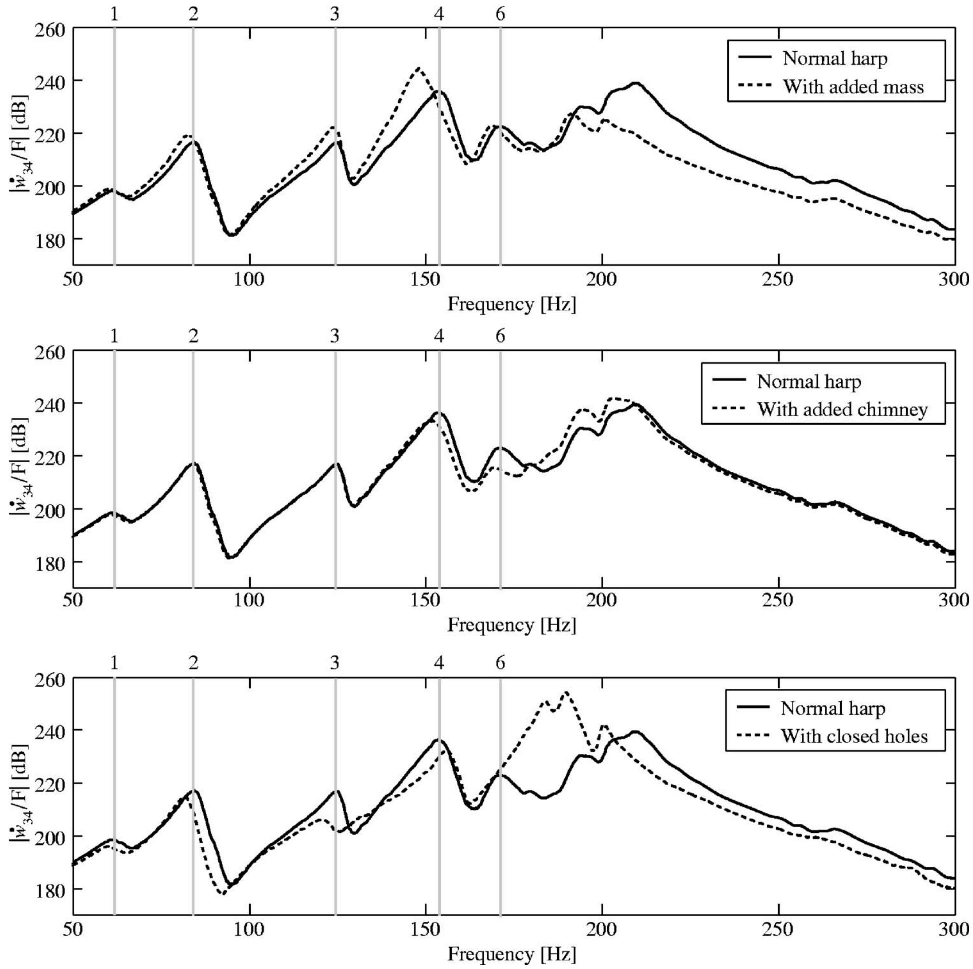

FIG. 8. Magnitude of the FRF $\dot{w}_{34}/F$ on the soundboard depending on four configurations: normal, mass added on the soundboard, chimney inserted in the lower hole, and with all holes closed. Numbers associated to vertical lines indicate the modal frequencies given in Fig. 4.

is no longer distinct. This result is also confirmed by the acoustic pressure measurement in front of the harp as shown in Fig. 7 when the sound holes are closed. This observation can be interpreted by the fact that the resonance of the open cavity does not exist anymore (see Sec. II B).

The second modification (2) consists of inserting a 2-cm-high chimney in the lower hole of the harp as shown in Fig. 9. This change induces an increase of the mass of the first air piston. Only two structural modes are affected: modes 4 and 6. This proves that these two modes are coupled to the fluid inside the cavity. The other modes are weakly coupled to the air cavity and do not participate in the acoustic response function as shown in Fig. 7.

The third modification (3) consists of adding a mass (m=200 g) on both sides of the central line of the soundboard as shown in Fig. 9. All eigenfrequencies of structural modes are lowered but modes 4 and 6 more than modes 1, 2, and 3. This is probably due to the fact that the mass is located on the maximum displacement area of these two modes.

TABLE II. Resonance frequencies for the first six modes according to four configurations of the instrument: normal (f), mass loaded on the soundboard (f_m), chimney inserted in the lower hole (f_c), and holes closed (f_{cl}).

	Resonance frequencies (Hz)				Deviations (Hz)		
Modes	f	f_m	f_c	f_{cl}	$f-f_m$	$f-f_c$	$f-f_{cl}$
1	61.5	60	61.5	60	1.5	0	1.5
2	84.5	82	84.5	81.5	2.5	0	3
3	124.5	123.5	124.5	120.5	1	0	4
4	153.5	148.5	152	156	5	1.5	2.5
6	172	168.5	169	…	3.5	3	…

B. Discussion

The most important effects of the modifications (1), (2), and (3) on modes 4 and 6 can be summarized as follows: on one hand, when sound holes are closed, mode 6 disappears. On the other hand, when the mass of the soundboard is increased, the eigenfrequency of mode 4 is lowered whereas the eigenfrequency of mode 6 is nearly stable. When the mass of the air pistons is increased, the eigenfrequency of mode 6 is lowered whereas the eigenfrequency of mode 4 undergoes smaller modifications. By considering these experimental results, it can be concluded that modes 4 and 6 involve a coupling between the bending motion of the soundboard mode and the oscillation of the air piston. These two modes can respectively be labeled, with the common notation, T1 and A0.

The fact that the A0 mode is present in the instrument's response clearly depends on the modal density and on the damping coefficients of the acoustical and structural modes. For some configurations, these parameters are such that the contribution of the A0 mode can be a minor one.[17] In our configuration, although sound holes are designed to ease the

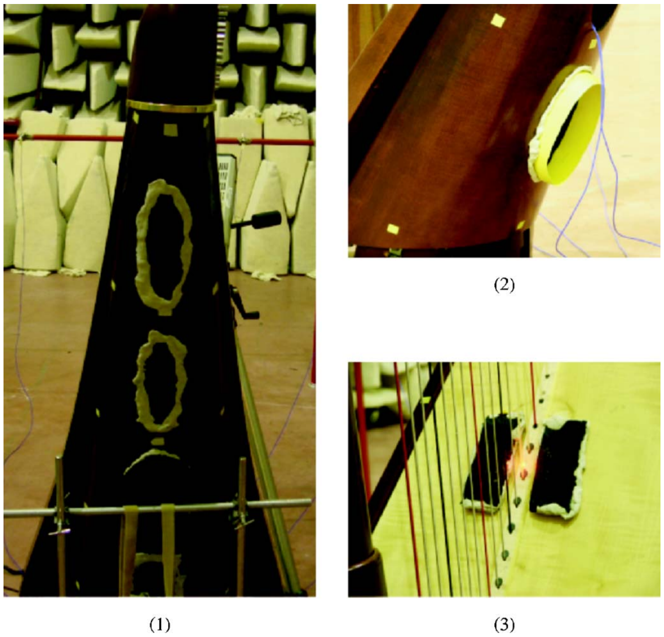

FIG. 9. (Color online) Different configurations of the modified instrument: (1) with all holes closed, (2) with chimney inserted in the lower hole, and (3) with mass added on the soundboard.

string mountings, they are found to have a significant influence on the vibroacoustic response of the concert harp. This is confirmed by the measurements of the far field acoustic pressure performed with opened and closed sound holes, as shown in directivity patterns (Fig. 6) and in the frequency response functions (Fig. 7).

Contrary to the violin and to the guitar, the A0 mode is above rather than below the frequency of the lowest acoustically significant structural mode T1. The concert harp does not take advantage of the soundbox resonance to increase its sound radiated in low frequencies, below the T1 mode.

Since these two modes, T1 and A0, are dominant in the low-frequency range, the response of the instrument can be approximated by using a two degrees of freedom oscillator model in which one degree is due to the soundboard and the other one to the fluid inside the cavity as it was done for the guitar.[4]

IV. CONCLUSION

This paper deals with the vibroacoustic behavior of a concert harp in the low-frequency range. The nature of the modes of the soundbox coupled to the internal fluid is investigated.

A classic experimental modal analysis has permitted the identification of six modes in the frequency range 24–181 Hz. Since the modal density increases with the frequency, mode identification at higher frequencies was not possible. Among the six identified modes, four correspond to global motions of the soundbox, which do not induce a change in the volume of the cavity and are thus weakly coupled to the internal acoustic field. These modes, which mostly depend on the characteristics of the connection of the soundbox to the arm and to the bottom of the pilar, lead to the weakening of the acoustic radiation. The two remaining modes, called T1 and A0, play an important acoustic role and have the following characteristics. (1) They are associated to coupled motions of the bending vibration of the soundboard and to the oscillations of the air pistons located in the sound holes. They correspond to the first two modes of a Helmholtz resonator with yielding walls. The labels T1 and A0 were used for the guitar and the violin for which this Helmholtz effect is known. (2) Modes T1 and A0 lead to important acoustic radiation: the acoustic pressure radiated by the harp takes high values in the range 140–230 Hz and the first two peaks of the pressure amplitude correspond to the resonance frequencies of T1 and A0. (3) The mode shapes of T1 and A0 are such that the displacement of the air pistons located in the five holes are all in phase. For T1, the displacements of these pistons are approximately in phase with the bending displacement of the soundboard. For A0, these motions are approximately out of phase. (4) Contrary to the violin and to the guitar, the A0 mode is above rather than below the frequency of the lowest acoustically significant structural mode T1. Thus, the concert harp does not take advantage of the soundbox resonance to increase its sound radiated in low frequencies. However, the study reveals the importance of the contribution of mode A0 in the response of the instrument, confirming the importance of the coupling between the soundboard and the cavity. This result is valid for the studied harp: *Atlantide Prestige* concert harp. Future works may concern others harps with different characteristics on which the eigenfrequencies of modes A0 and T1 depend: cavity volume, sound holes sizes, and soundboard material.

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